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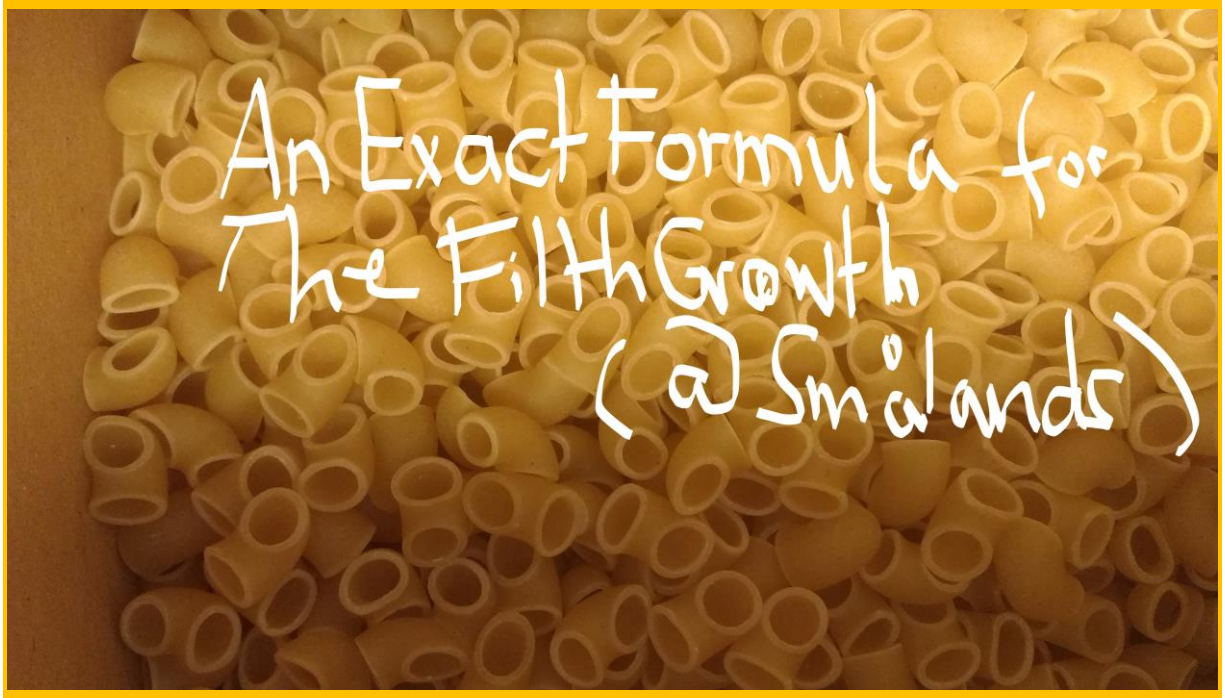

What 9 Macaroni Say about the Environmental Crisis

This observation and early formalization of a tragedy of the commons at a kitchen at the Smålands fraternity, was carried out around 2005 at a meeting about the worrying and annoying mess that rapidly accumulated in the basin and elsewhere in the (common) dorm kitchen. The theory was not very well received.

A tragedy of the commons with altruistic and fair-minded individuals – The example of macaroni and litter growth in the common dorm kitchen

Background: Smålands nation is a very politically correct fraternity where all individuals have or are expected to have strong socialistic morale, so how come many of the *commons* in the dorms are mismanaged? How can we explain the litter in the kitchen when individuals in the dorms have such strong sense of cooperation and fairness? Is it possible that these norms can be consistent with the conspicuously messy kitchens?

In this paper I derive an exact, explicit formula for the maximal growth of macaroni in the kitchen basin, implied by assumptions including a very strong and kind version of fairness and willingness to take care of the common good. This example can be applied to other discrete objects that can form approximately homogenous piles of dirt.

Assumptions:

(i) Litter/macaroni $m \in \{0,1,\dots,n\}$ and there exists I individuals in the community $i \in \{0,1,\dots,I\}$ and there are T periods $t \in \{0,1,\dots,T\}$

(ii) Each individual i may accidentally drop an arbitrary nr. Of macaroni in the basin at time t which is defined as the *contribution* c_{it}.

(iii) The sum of macaroni in the basin forms a homogenous pile of litter which can only be removed completely of left to grow, the pile at time t is p_t and is the sum of all earlier contributions not picked up.

(iv) Due to strong sense of fairness and altruism among the individuals, these individuals will pick up the pile of litter whenever their accidental contribution to the pile c_{it} makes up ½ of the total sum of macaroni/litter. If the contribution to the pile made by an individual is less than ½ of the total amount, then they will leave the crap because then it is perceived as unfair to pick up the pile.

Initially everything is fine, tidy and clean. But then an *exogenous shock occurs* and someone accidentally leaves (a contribution to the pile) $c_{i0} \geq 2$ macaroni in the basin.

The macaroni dynamics

Assume c_{i0}=2, i.e. someone accidently drops two macaroni without picking it up. Then these two form one pile according to (iiI) and the initial pile is then $p_0 = 2$.

Assume now someone drops at least 2 macaroni at time 1, i.e. c_{i1}=2, then the process stops because the individual will pick up the mess according to (iv) ($P_1 = P_0 + C_{i1}$ =2+2=4 and ½ *4=2=c_{i1}).

Assume now the accident is C_{i1}=1, now the pile is $P_1 = P_0 + C_{i1} = 2 + 1 = 3$ but then $\frac{C_{i1}}{P_1} = 1/3 < 1/2$, so $P_1 = 3$. Clearly if $C_{i2} = 3$, then the process stops & it's clean again, but anything below P_1 adds up to a greater pile. According to (i)+(iv) we see that in general, the pile will grow if

$C_{it} \leq P_{t-1} - 1$ and grows maximally in equality.

We see that the maximal growth is $\Delta P_t = C_{it}^{max} = P_{t-1} - 1 \leftrightarrow P_t = 2P_{t-1} - 1$

Now $P_t = 2P_{t-1} - 1 \leftrightarrow P_t - 2P_{t-1} = -1$, we solve the homogenous equation:

$$Cr^t - 2Cr^{t-1} = 0 => r = 2, so\ X_h = C2^t$$

The particular equation is, setting p=c: c-2c=-1 ⇔c=1, which gives $X_p = 1$ so the solution is:

$$X = X_p + X_h = C2^t + 1\ and\ with\ P_0 = 2\ this\ becomes: C + 1 = 2\ \rightarrow C = 1$$

Which gives:

$$P_t = 2^t + 1$$

Which generates an exponential growth of filth, the sequence is: 2, 3, 5, 9, 17, 33….

Conclusions

I have shown that the tragedy of the commons prevails even in the presence of strong feelings for the common good and fairness norms at an arbitrary Smålands dorm. These conclusions can be carried over to other similar scenarios, especially other dorms as the ones at Blekingska.

Extensions:
Stochastic model

Caps on the pan size, which is worse a big macaroni pan or a small (a small!)?

Other norms and punishment